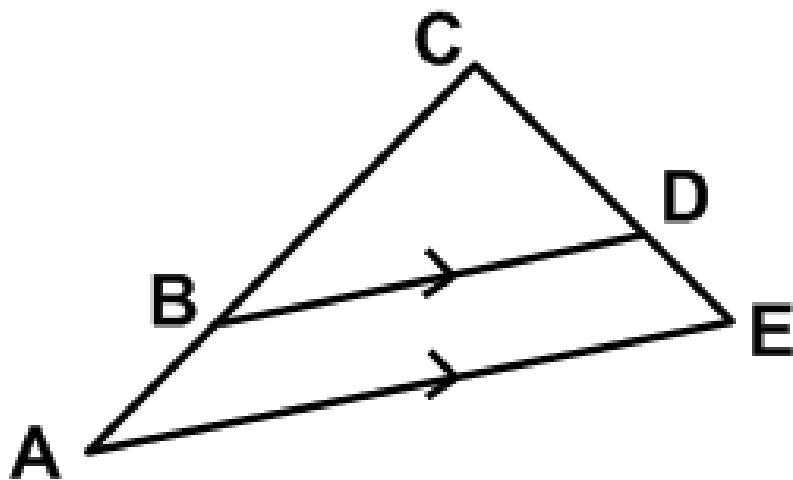# 6 - 4
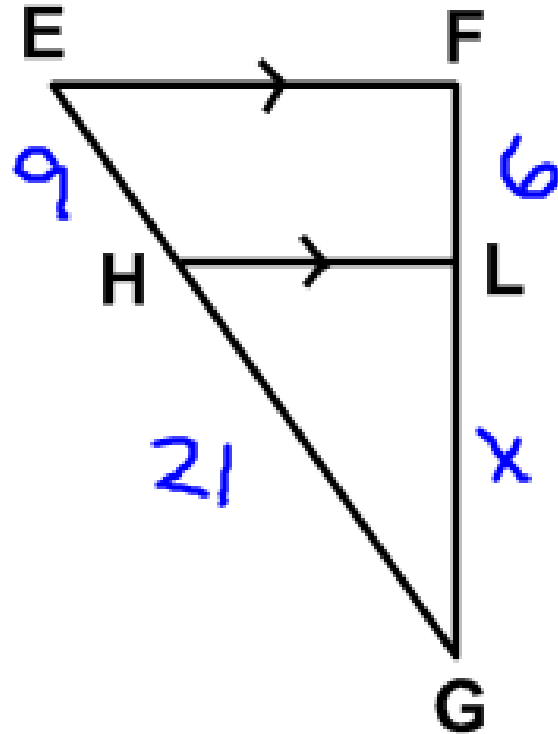# Parallel Lines and Proportional Parts (Day One)

Theorem 6.4:
(Triangle Proportionality Theorem)

If a line is _parallel_ to one side of a triangle and _intersects_ the other two sides in two distinct points, then it separates these sides into segments of _proportional lengths_.

$$\frac{CB}{BA} = \frac{CD}{DE}$$

Ex: In $\triangle EFG$, $\overline{HL} \parallel \overline{EF}$, EH = 9, HG = 21, and FL = 6. Find LG.

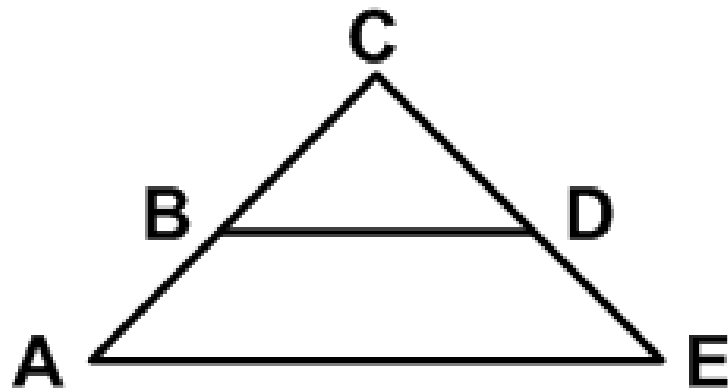$$\frac{9}{21} = \frac{6}{x}$$

$$\frac{9x}{9} = \frac{126}{9}$$

$$x = 14$$

$$\boxed{LG = 14}$$
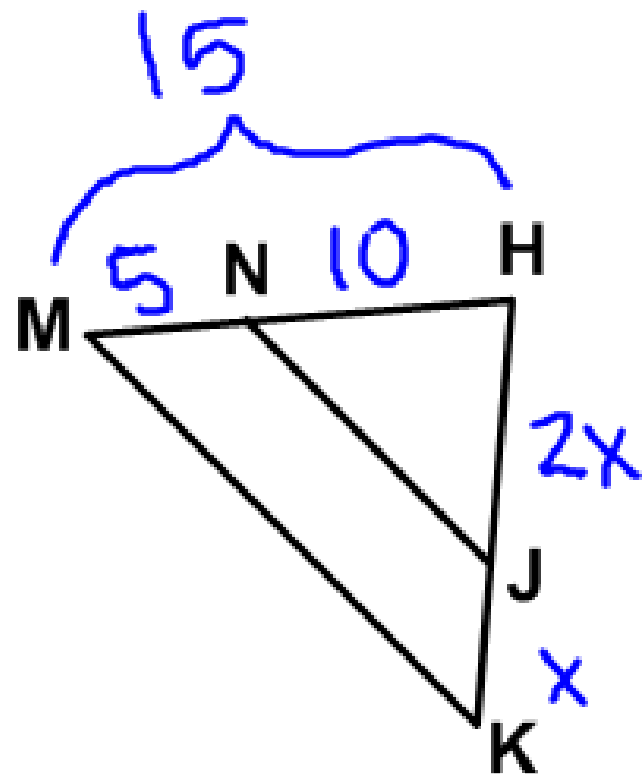
Theorem 6.5:
(Converse of the Triangle Proportionality Theorem)

If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional length, then the line is parallel to the third side.
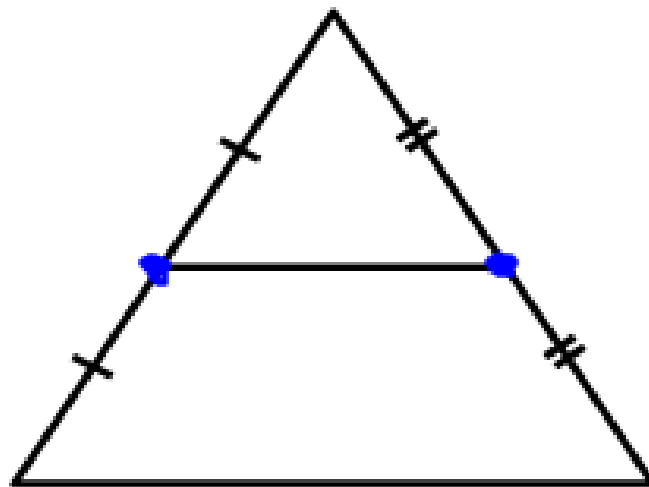
Ex: In $\triangle$HKM, HM = 15, HN = 10, and $\overline{HJ}$ is twice the length of $\overline{JK}$. Determine whether $\overline{NJ} \parallel \overline{MK}$.
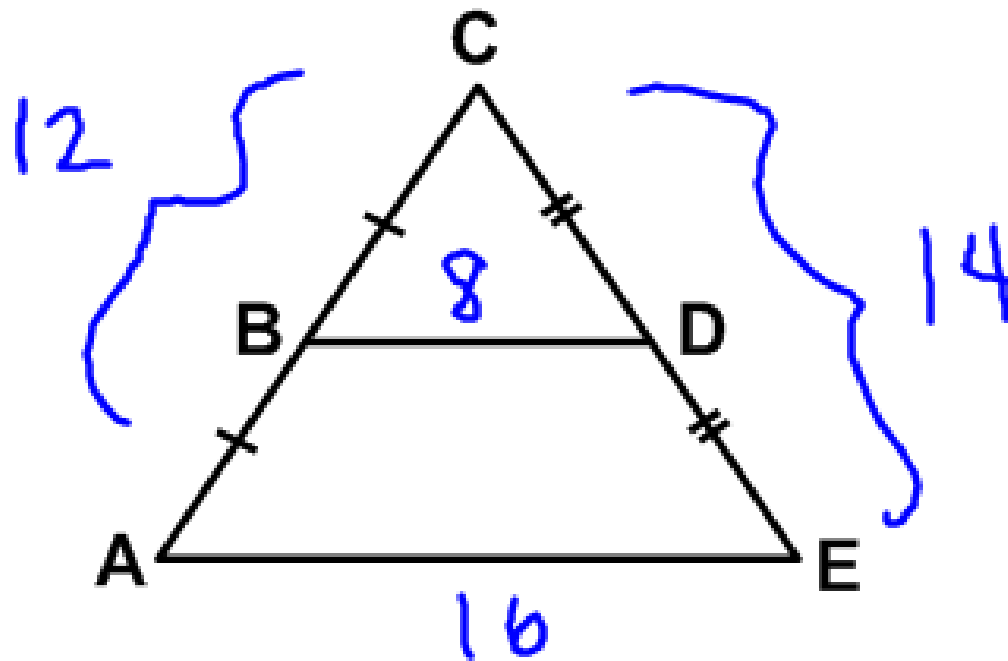
Yes

15

M 5 N 10 H

2x

J

x

K

midsegment: segment whose endpoints are the midpoints of 2 sides of a △
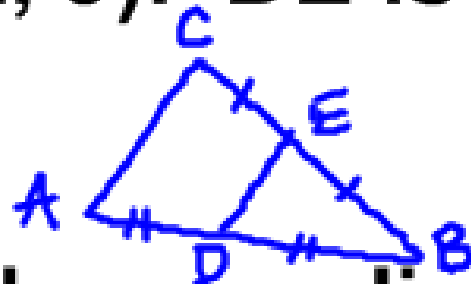
Theorem 6.6
(Triangle Midsegment Theorem)

A midsegment of a triangle is _parallel_ to one side of the triangle, and its length is _$\frac{1}{2}$ the length of that side._

12

8

B   D

14

A   E

16

Ex: $\triangle$ABC has vertices A(- 4, 1), B(8, - 1), and C(- 2, 9). $\overline{DE}$ is a midsegment of $\triangle$ABC.

a.) Find the coordinates of D and E.

$$D : \left(\frac{-4+8}{2}, \frac{1+-1}{2}\right) = (2,0)$$

$$E : \left(\frac{8+-2}{2}, \frac{-1+9}{2}\right) = (3,4)$$

Ex: $\triangle$ABC has vertices A(- 4, 1), B(8, - 1), and C(- 2, 9). $\overline{DE}$ is a midsegment of $\triangle$ABC.
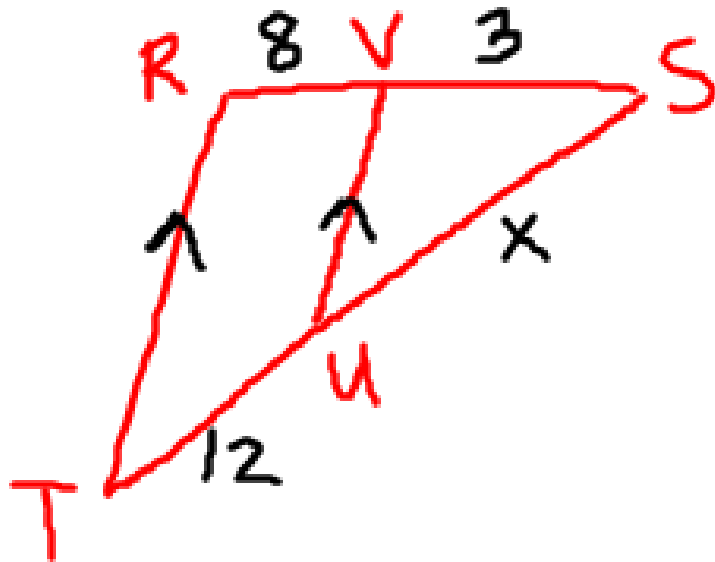
b.) Verify that $\overline{AC}$ is parallel to $\overline{DE}$.

$$m\overline{AC} = \frac{1-9}{-4+2} = \frac{-8}{-2} = 4$$

$$m\overline{DE} = \frac{0-4}{2-3} = \frac{-4}{-1} = 4$$

On your own...

1.) In $\triangle RST$, $\overline{RT} \parallel \overline{VU}$, $SV = 3$, $VR = 8$, and $UT = 12$. Find SU.

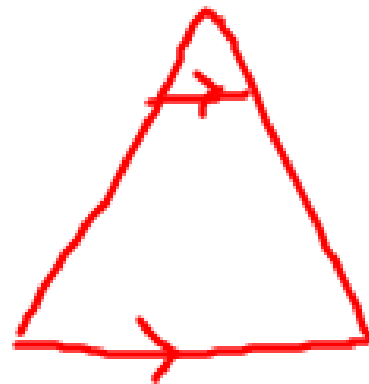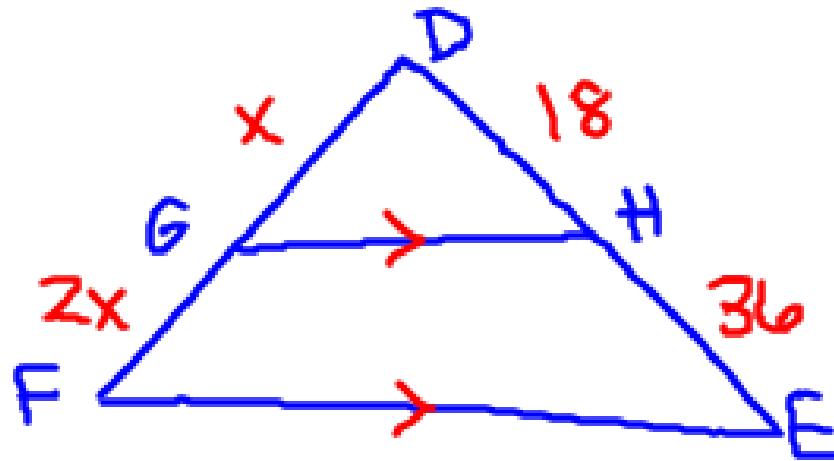$$\frac{3}{8} = \frac{x}{12}$$

$$36 = 8x$$

$$\frac{36}{8} = x$$

$$\frac{18}{4}$$

$$9\frac{1}{2}$$

$$x = \frac{9}{2}$$

$$x = 4\frac{1}{2}$$

$$x = 4.5$$

2.) In $\triangle DEF$, DH = 18, HE = 36, and DG = $\frac{1}{2}$ GF. Determine whether $\overline{GH} \parallel \overline{FE}$.
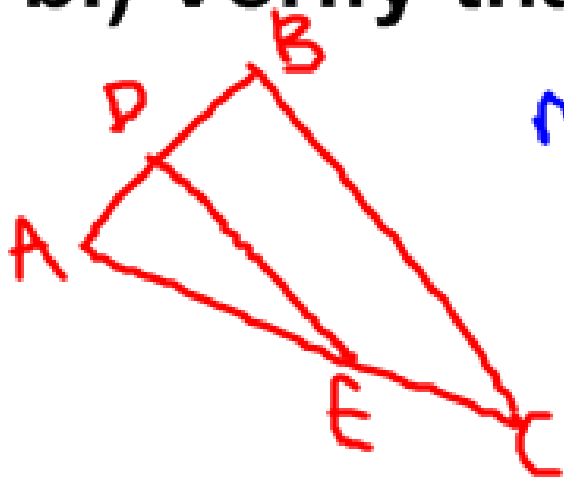
3.) $\triangle ABC$ has vertices A(-2, 2), B(2, 4), and C(4, -4). $\overline{DE}$ is a midsegment of $\triangle ABC$.

a.) Find the coordinates of D and E.

$D(0, 3)$ $E(1, -1)$

b.) Verify that $\overline{BC} \parallel \overline{DE}$.

$m\overline{BC} = \dfrac{8}{-2} = -4$

$m\overline{DE} = \dfrac{4}{-1} = -4$